

Robust synchronization of chaotic systems with fractional order with adaptive fuzzy sliding mode control

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Abstract— In this paper, the issue of chaos control and synchronization problems of two systems of fractional order with adaptive fuzzy sliding model has been addressed. According to Lyapunov stability theory and sliding mode control law, control will be extracted for system synchronization. Then, control signal is generated and with the signal of sliding surface (S) and changes of sliding surface (dS/dt), it is selected as data of a fuzzy neural network. After creating fuzzy system, this control system was designed as controller. Finally, numerical simulation of synchronization of two chaotic systems with fractional order and the robustness of this controller against disturbance noise and uncertainty are shown.

Index Terms— chaos, fractional order systems, Lyapunov stability, sliding mode control, adaptive control, fuzzy control

1 INTRODUCTION

Fractional differential calculus relates back to about 300 years ago. However, its applications in physics and engineering have started in recent decades. Many systems in interdisciplinary areas can be modeled by fractional order derivatives [1]. Controlling and synchronizing chaotic systems have been one of the most interesting subjects in recent years and attracted scientists' attention. For example, in [2], synchronization of the integrated fractional order chaotic systems has been studied. [4] Presents control based on active sliding mode controller for synchronization of fractional order chaotic system. [5] Uses fractional Routh-Horowitz conditions for controlling fractional order chaos in Duffing-Vandepol system. In [6], a smart fractional sliding surface has been defined and a sliding controller has been studied for a nonlinear system. The new fractional order hyper-chaotic system has been presented in [7] and designed by placing pole for synchronizing a class of non-linear fractional order systems. [8] Studies the coordination between fractional order chaotic systems. A simple but efficient method has been presented in [9] for controlling the fractional chaotic system using T-S fuzzy model and an adaptive regulation mechanism. In [16] investigates the synchronization of coupled chaotic systems with many equilibrium points. By addition of an external switching piecewise-constant controller, the system changes to a new one with several independent chaotic attractors in the state space. Then, by addition of a nonlinear state feedback control, the chaos synchronization is presented. In [17] in this paper, the synchronization problem for fractional-order chaotic systems is investigated. An adaptive observer-based slave system is designed to synchronize a given chaotic master system.

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2. SYSTEMS WITH FRACTIONAL DERIVATIVES

Despite of complications of differential calculus, recent advancements in chaotic systems and the close relationship between fractals and fractional calculus have grown interests in applying it. Fractional calculus has a wider range than correct derivative. If we use fractional order instead of correct order derivative or integral, we should use fractional calculus for solving derivative and fractional integral.

$${}_a D_t^q = \begin{cases} \frac{d^q}{dt^q} q > 0 \\ lq = 0 \\ \int_a^t (d\tau)^{-q} q < 0 \end{cases} \quad 2-1$$

Derivative-integrator operator is shown by ${}_a D_t^\alpha$. This operator is a sign for taking derivative and fractional.

Grunwald-Letnikov, Riemann-Liouville, and Caputo are definitions that are applied for fractional derivatives.

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{[t-\alpha/h]} (-1)^j \binom{\alpha}{j} f(t-jh),$$

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Its Laplace transform is as follows:

$$\begin{aligned} d^q x_1 &= x_2 \\ d^q x_2 &= x_3 \\ d^q x_3 &= -cx_1 - bx_2 - ax_3 - x_1^2 \\ a &= 1.2; b = 2.92; c = 6; \end{aligned} \tag{4-2}$$

$$\int_0^\infty e^{-st} {}_0D_t^\alpha f(t) dt = s^{-\alpha} F(s) \quad \alpha \leq 0$$

$$\int_0^\infty e^{-st} {}_0D_t^\alpha f(t) dt = s^{-\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(t) \Big|_{t=0}$$

$$n - 1 < \alpha \leq n \in N(2)$$

3. Problem Description

Synchrony has a Greek root meaning to share the common time. The principle meaning of this word has been kept in ordinary application referring to settlement or affinity. Analysis of synchronization in dynamic systems has been a subject of considerable debate in physics as an important matter. The origin of this phenomenon relates back to 17th century when the second phase of pendulum clock hanging from a point and there was a weak coupling between them coordinate with each other. Later on, other samples of such phenomena were also observed. Recently, studies have extended to chaotic systems. As we know, chaotic systems are sensitive to primary conditions. Because of this property, these systems oppose to synchronization by nature. Even two totally similar systems starting to work with trivial differences gradually lose their coordination over times.

4. DESIGNING THE SLIDING MODE TO SYNCHRONIZE A CHAOTIC SYSTEM GENESIO_TESI WITH FRACTIONAL ORDER

Consider a genesio tesi chaotic system with fractional order q ($0 < q < 1$). It is described by a fractional differential equation as follows:

$$\begin{aligned} d^q x_1 &= x_2 \\ d^q x_2 &= x_3 \\ d^q x_3 &= -cx_1 - bx_2 - ax_3 - x_1^2 \end{aligned} \tag{4-1}$$

System (4) is chaotic when $a = 1.2$, $b = 2.92$, $c = 6$,

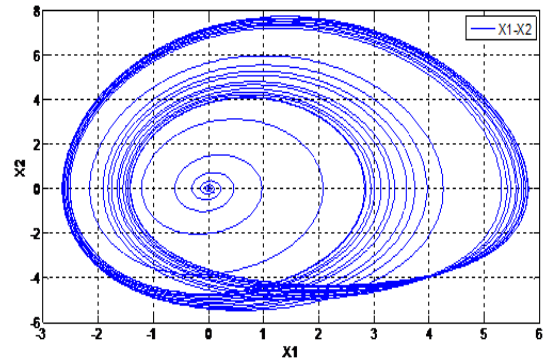


Figure1. Chaotic attractor of x1-x2

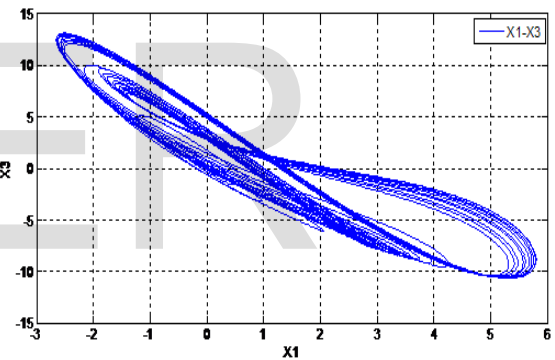


Figure2. Chaotic attractor of x1-x3

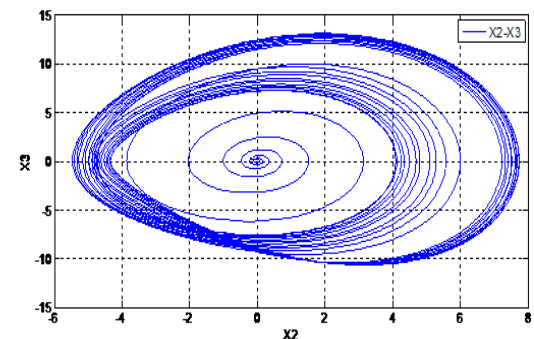


Figure3. Chaotic attractor of x2-x3

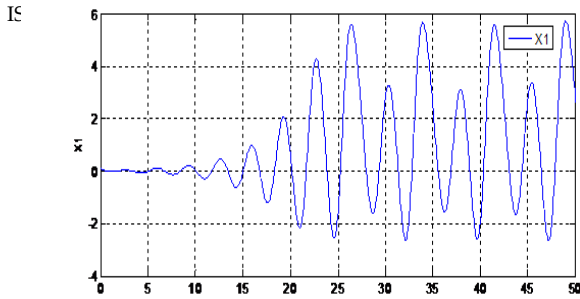


Figure4.Response of X1 state

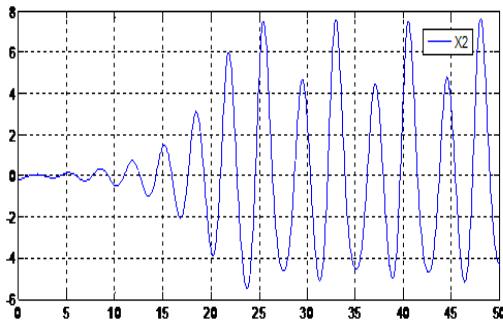


Figure5.Response of X2 state

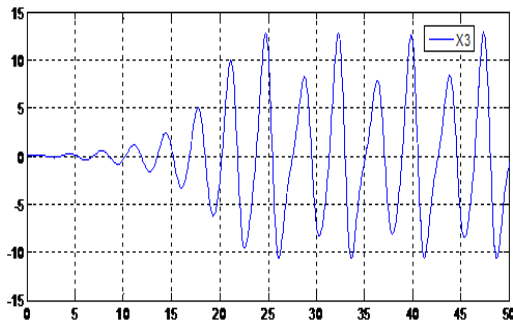


Figure6.Response of X3 state

Master system

$$\begin{aligned} d^q x_1 &= x_2 \\ d^q x_2 &= x_3 \\ d^q x_3 &= -cx_1 - bx_2 - ax_3 - x_1^2 \end{aligned} \tag{4-3}$$

Slave system

$$\begin{aligned} d^q y_1 &= y_2 + u_1 \\ d^q y_2 &= y_3 + u_2 \\ d^q y_3 &= -cy_1 - by_2 - ay_3 - y_1^2 + u_3 \end{aligned}$$

Synchronization means finding the control signal of $u(t) \in R^3$ to close the state of the follower system to the based state. To reach this goal, we can define the dynamic error of coordination as follows:

$$\begin{aligned} D^q e_1 &= e_2 + u_1 \\ D^q e_2 &= e_3 + u_2 \end{aligned} \tag{4-5}$$

$$D^q e_3 = -ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 + u_3$$

We have $e_i = y_i - x_i$ and

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{4-6}$$

And

$$G(x_1, x_2) = \begin{bmatrix} 0 \\ 0 \\ y_1^2 - x_1^2 \end{bmatrix} \tag{4-7}$$

To simplify, we substitute that the linear part of the follower system with Matrix $A = A_2$.

The main aim is to design controller $u(t) \in R^3$ to have:

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0 \tag{4-8}$$

According to the method of designing active controller [6], the nonlinear part of dynamic error is eliminated by choosing the input vector:

$$u(t) = H(t) - G(x_1, x_2) \tag{4-9}$$

The error of system (11) is rewritten as bellow:

$$D_t^q e = A_e e + H(t) \tag{4-10}$$

The equation (16) describes the error dynamic by a new definition for the input of controller $H(t)$. In active sliding mode controller, $H(t)$ is developed based on sliding mode rules:

$$H(t) = Kw(t) \tag{4-11}$$

$K = [k_1, k_2, k_3]^T$ is the fixed gain vector and $w(t) \in R^3$ is the controller input which is defined as bellow:

$$w(t) = \begin{cases} w^+(t) & s(e) \geq 0 \\ w^-(t) & s(e) < 0 \end{cases} \tag{4-12}$$

$S = s(e)$ is the switching surface where dynamics are placed in its favorable zone. As a result, dynamic error is:

$${}_0 D_t^q e = A_e + Kw(t) \quad 4-13$$

According to what was mentioned before, the good sliding controller is achieved based on developing the theory of sliding mode control [10-5].

5. Developing the Sliding Surface

The sliding surface can be determined as bellows:

$$s(e) = Ce \quad 5-1$$

$C = [c1, c2, c3]$ is the fixed vector. By solving $\dot{s}(e) = 0$, which is the required condition, the equivalent control is achieved. $S \in 0$ is a condition for the state curve to remain in the switching level. Thus, in sliding mode controller, the two following conditions should be met:

$$s(e) = 0, \quad \dot{s}(e) = 0 \quad 5-2$$

$$\dot{s}(e) = \frac{\partial s(e)}{\partial e} \dot{e} \quad 5-3$$

$$\frac{\partial s(e)}{\partial e} {}_0 D_t^{1-q} ({}_0 D_t^q e) = C {}_0 D_t^{1-q} [A_e + Kw(t)] = 0$$

$${}_0 D_t^{1-q} w(t) = -(CK)^{-1} CA ({}_0 D_t^{1-q} e(t)) \quad 5-4$$

$$w_{eq}(t) = -(CK)^{-1} CA e(t) \quad 5-5$$

$${}_0 D_t^{1-q} e = (I - K(CK)^{-1}C)Ae \quad 5-6$$

6. Developing Sliding Mode Controller

In designing the constant, we consider the relative convergent speed. Accordingly, the condition of reaching is chosen as follows:

$${}_0 D_t^{1-q} s = -p \operatorname{sgn}(s) - rs \quad 6-1$$

$\operatorname{Sgn}(\cdot)$ is the sign function. Gains $p > 0$ and $r > 0$ are determined to meet the sliding conditions and the sliding mode movement then occurs.

According to equations 19 and 20 :

$${}_0 D_t^{1-q} s = C {}_0 D_t^q e = C [A_e + Kw(t)] \quad 6-2$$

Now from equations 2-5 and 2-6, the input control is determined:

$$w(t) = -(CK)^{-1} [C(rI + A)e + p \operatorname{sgn}(s)] \quad 6-3$$

$$u(t) = K (-(CK)^{-1} [C(rI + A)e + p \operatorname{sgn}(s)]) - G \quad 6-4$$

7. Numerical simulation

Here, the numerical simulation results obtained by MATLAB for chaotic systems with similar order `gensio_tesi`:

7-1-Synchronization of two fractional order systems `gensio_tesi`

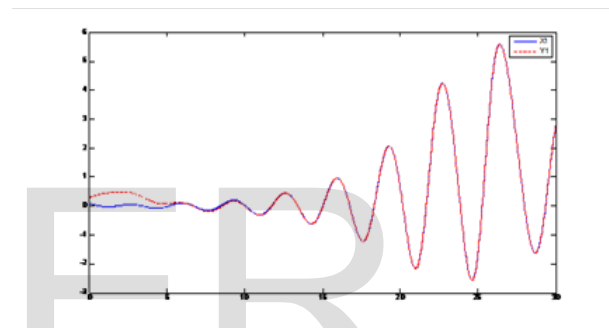


Figure 7. Response of x1 and y1 after imposing sliding mode control

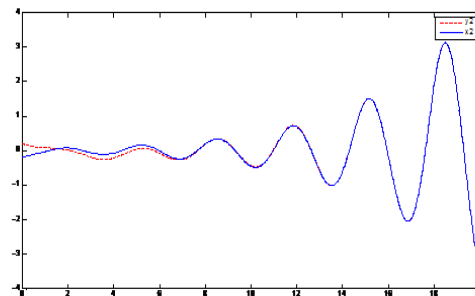


Figure 8. Response of x2 and y2 after imposing sliding mode control

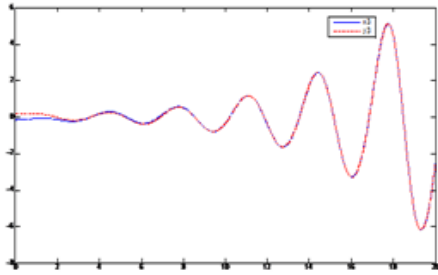


Figure 9. Response of x3 and y3 after imposing sliding mode control

From the conducted design in the first part, we receive some signals in the workplace and we use them as network data of adaptive Nero fuzzy.

- u1(t)=Control signal of first mode
- u2(t)=control signal of second mode
- u3(t)=control signal of third mode
- s(t)=sliding surface
- ds(t)=the changes of sliding surface

5 vectors are stored in the workplace, the purpose of designing is to design 3 fuzzy system by using ANFIS to replace u1(t), u2(t), u3(t).

To train the first ANFIS, we should create a matrix as following.

$$\text{data} = [s \quad ds \quad u1]$$

7-1

In ANFIS training, the last column of output must necessarily be desired.

From the existing data, we have used 70% of them as training data and 30% of them as test data. It is better that in selecting train data and test, we select them randomly and it means we should choose them randomly from the total data.

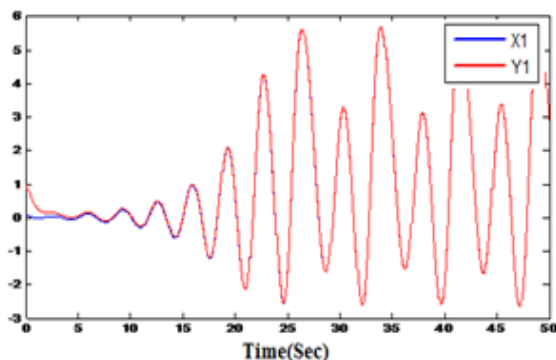


Figure 10. the response of x1 and y1 after imposing adaptive fuzzy control

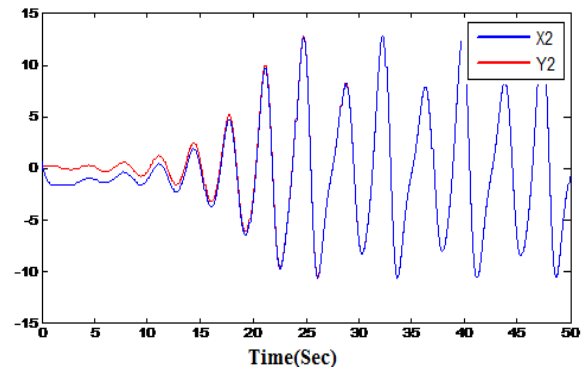


Figure 11. the response of x2 and y2 after imposing adaptive fuzzy control

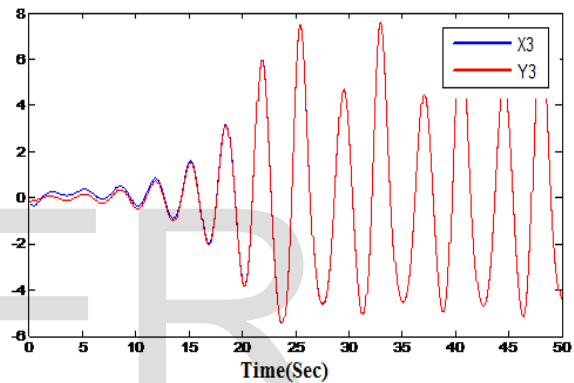


Figure 12. the response of x3 and y3 after imposing adaptive fuzzy control

7-2-The effect of uncertainty:

At this stage, we impose previous controller to the system with uncertainty.

Uncertainty was assumed as follows:

$$a = a \pm \Delta a$$

$$\Delta a = \% 10a$$

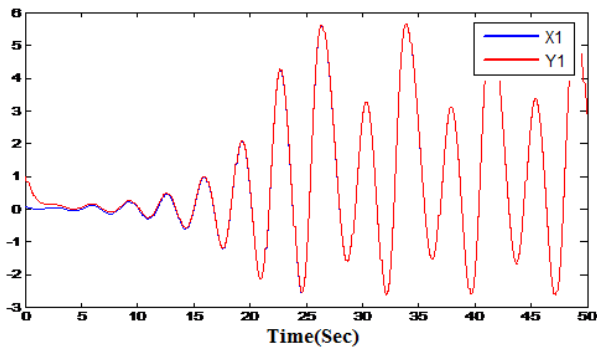


Figure 13. the response of x1 and y1 after imposing adaptive fuzzy control with uncertainty

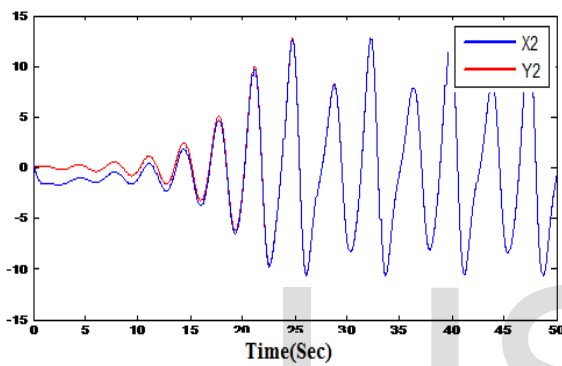


Figure 14. the response of x2 and y2 after imposing adaptive fuzzy control with uncertainty

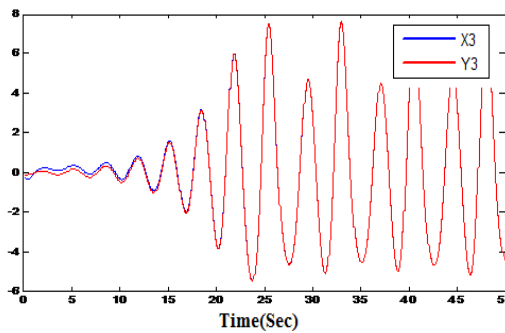


Figure 15. the response of x3 and y3 after imposing adaptive fuzzy control with uncertainty

In figures 13-14-15, the system response is shown with uncertainty. As it can be seen from figures, fuzzy sliding mode controller is entered to system with uncertainty and it can do the synchroni-

zation, well. This means that this controller is resistant to the imposed uncertainty.

7-3-The effects of disturbance:

At this point, we logged an external chaos signal to system.

$$D=0.2\sin t$$

This chaos is imposed to the third mode of master system.

$$d^q X_3 = -a * x_1 - b * x_2 - c * x_3 + x_1 * x_1 + d;$$

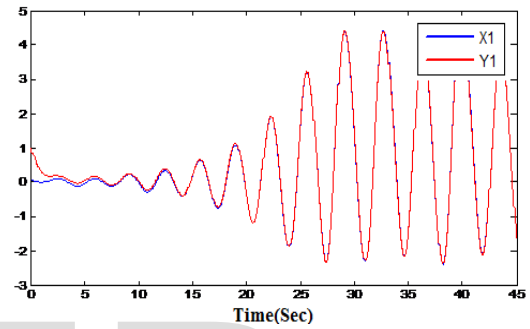


Figure 16. The response of x1 and y1 after imposing adaptive fuzzy control with chaos

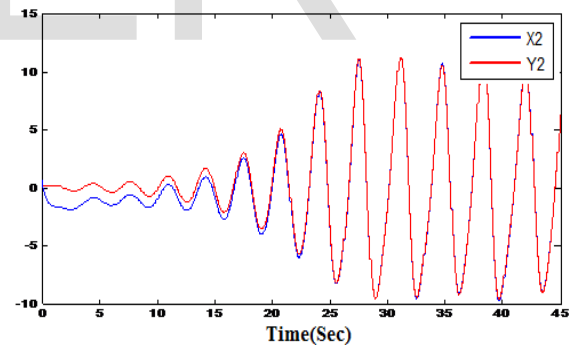


Figure 17. the response of x2 and y2 after imposing adaptive fuzzy control with chaos

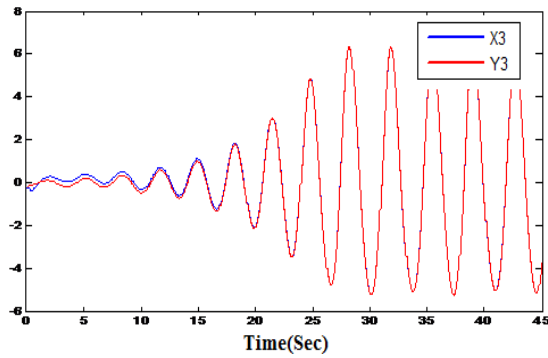


Figure 18. the response of x3 and y3 after imposing adaptive fuzzy control with chaos

The responses of system states are shown in figures 18-17-16. It can be seen that this controller is resistant to the external chaos and synchronization of these two fractional chaos systems is well down.

7-4-Noise effect:

At this point, we enter a noise signal to the system, entered noise is shown in figure 19.

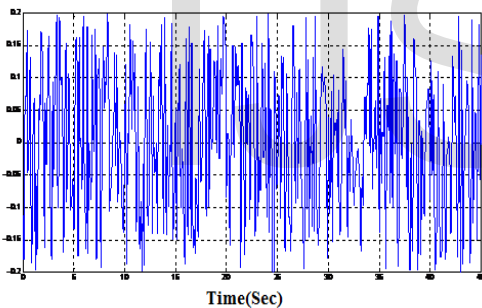


Figure 19. Entered Noise to system

This Noise is imposed to the third mode of master system.

$$d^q X_3 = -a * x_1 - b * x_2 - c * x_3 + x_1 * x_1 + N(t);$$

$$0 < t < t_{sim}$$

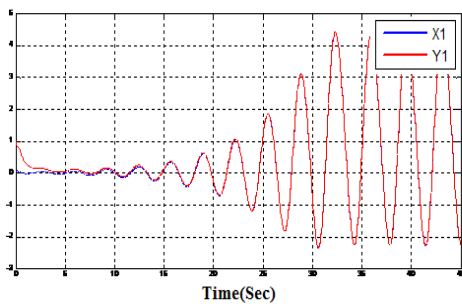


Figure 20. The response of x1 and y1 after imposing adaptive control fuzzy with noise

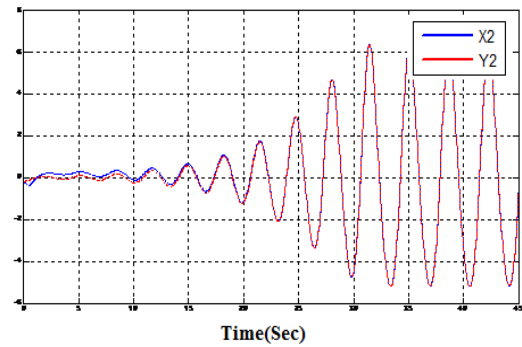


Figure 21. the response of x2 and y2 after imposing adaptive fuzzy control with noise

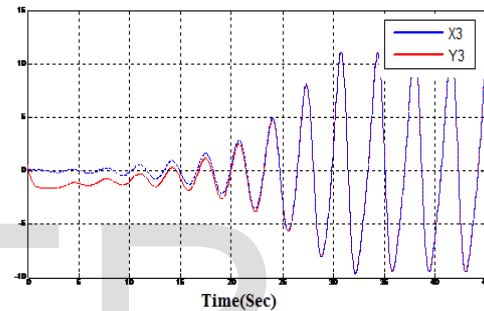


Figure 22. the response of x3 and y3 after imposing adaptive fuzzy control with noise

According to figure 19, a white noise with maximum range of 0.2 is logged into system, in figures 22-21-20, the system response is shown in presence of Noise.

7-5-The effect of changing the initial conditions in the system:

Designed controller is designed for the initial conditions, now we select the initial conditions of two systems as follows and we measure the effect of these factors on response.

$$X_0 = (1, -0.2, 0.6)$$

$$Y_0 = (0.2, 0.3, 0.2)$$

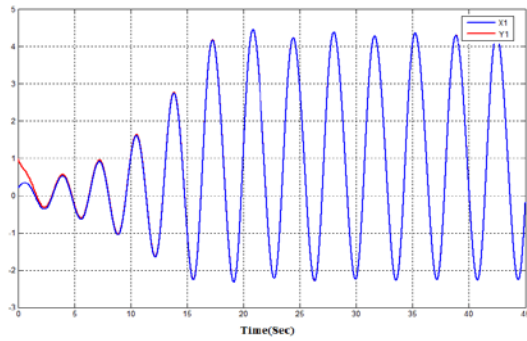


Figure 23. The response of x_1 and y_1 after imposing adaptive fuzzy control with changing initial conditions

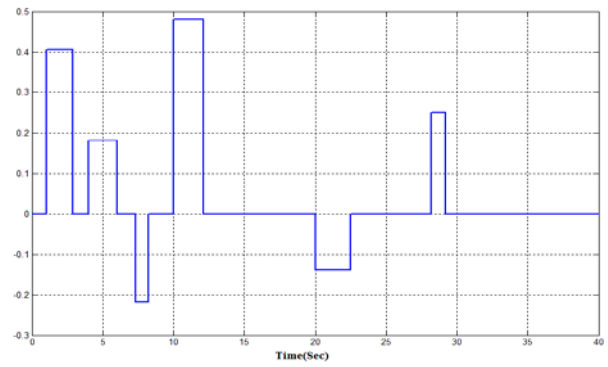


Figure 26. non-sinusoidal effect imposed to system

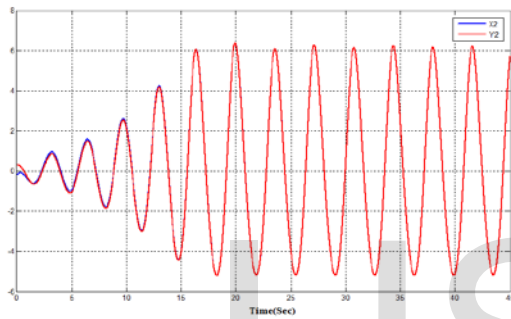


Figure 24. the response of x_2 and y_2 after imposing adaptive fuzzy control with changing initial conditions

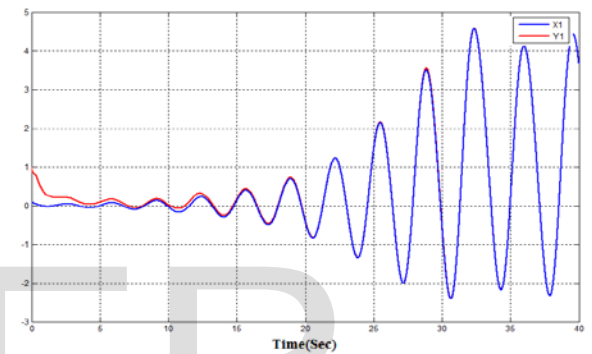


Figure 27. the response of x_1 and y_1 after imposing adaptive fuzzy control with non-sinusoidal chaos

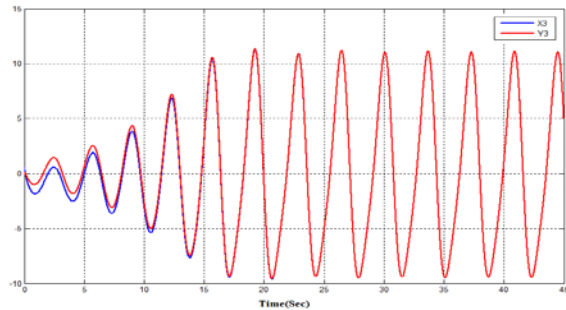


Figure 25. the response of x_3 and y_3 after imposing adaptive fuzzy control with changing initial conditions

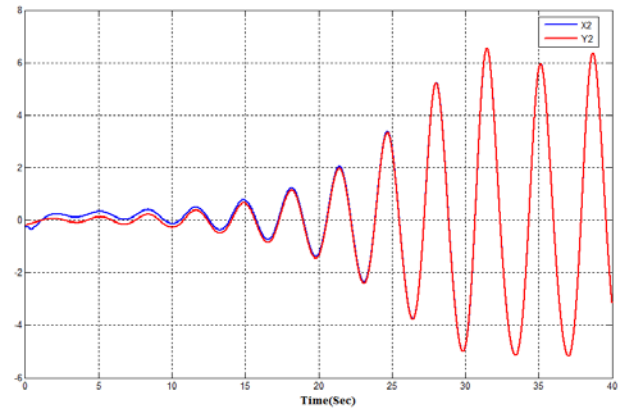


Figure 28. the response of x_2 and y_2 after imposing adaptive fuzzy control with non-sinusoidal chaos

7-6-The effect of Non-sinusoidal effect:

At this stage, one external chaos is entered as figure 26 to system.

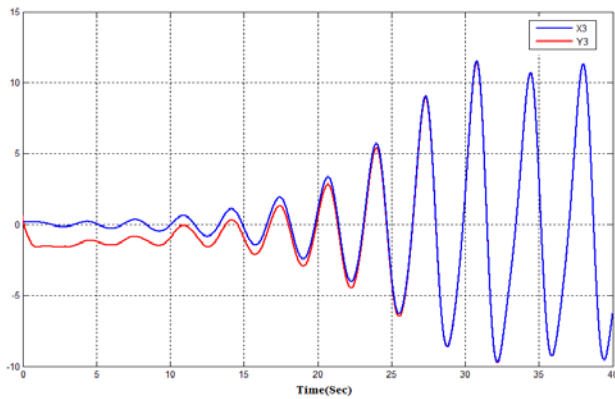


Figure 29. the response of x3 and y3 after imposing adaptive fuzzy control with non-sinusoidal chaos

To compare, the performance of this controller in different situations, the following criteria is used for evaluation.

$$MSE = \frac{1}{N} \sum_{i=1}^N e(i)^2$$

	MSE1	MSE2	MSE3
Fuzzy	0.0068	0.0054	0.1723
Changing initial conditions	0.0080	0.0056	0.1736
Uncertainty	0.0017	0.0156	0.5064
Non-sinusoidal disturbance	0.0163	0.0160	0.7427
Disturbance	0.0161	0.0158	0.7464
Noise	0.0163	0.0195	0.9142

Conclusion:

In this paper, chaos systems synchronization of fractional order is investigated by adaptive fuzzy control method. According to Lyapunov stability and control theory of sliding mode, a controller is designed for stabilization and synchronization of fractional order chaotic systems. In simulation, initial conditions of $X(0)=(0.1, -0.2, 0.2)$ and $Y(0)=(1, -0.2, 0.6)$ and $q=0.95$ and $c=6$ and $b=2.92$ and $a=1.2$ are included. Simulation results

are reported of the utility of this method for synchronization of chaotic and fractional order systems. The proposed controller shows an appropriate response against chaos and uncertainty

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